Course Title: Functional Analysis

Course Code: MATH-401

Course Type: Major Math

Prerequisites: Topology & Linear Algebra

Credit Hours: 3 (3 + 0)

Course Objectives: Students will be able to:

- Define and identify complete and separable metric spaces.
- Apply linear algebra and analysis techniques to function spaces.
- Analyze and work with normed linear spaces, Banach spaces, and Hilbert spaces, including their properties and applications.

Course Contents:

Metric Space: Review of metric spaces, Continuity of metric spaces, Convergence in metric spaces, Cauchy Sequences, Complete metric spaces, Dense sets and separable spaces, Proofs about completeness and separability of some classes of metric spaces, no-where dense sets, Baire category theorem.

Normed Spaces: Normed linear spaces, Banach spaces, equivalent norms, Convex sets, Quotient spaces, Linear operator, Finite dimensional normed spaces, Continuous and bounded linear operators, Linear functionals, Dual spaces.

Applications of Banach Spaces: Definition of fixed point and examples, Banach fixed point theorem, Classical Banach spaces, Distance measures.

Inner Product Spaces: Definition and examples, orthonormal sets and bases, Projections, Linear functionals on Hilbert spaces, Reflexivity of Hilbert spaces, The Riesz representation theorem, Annihilators and orthogonal complements, Direct decomposition.

Recommended Books:

1. Axler, S., *Measure, Integration & Real Analysis*, Graduate Texts in Mathematics, Springer, 2020.
2. Balakrishnan, A. V., *Applied Functional Analysis*, Springer-Verlag,2nd edition, 1981.
3. Conway, J. B., *A Course in Functional Analysis*, Springer-Verlag, 2nd edition, 1997.
4. Kreyszig, E., *Introduction to Functional Analysis with Applications*, John Wiley & Sons, Inc., 2004.
5. Yosida, K., *Functional Analysis*, Springer-Verlag, 5th edition, 1995.

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